Abelian and Center Vortex Condensation in SU(3) Lattice Gauge Theory

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We study the condensation of Abelian and Center vortices in SU(3) lattice gauge theory at finite temperature. We find that both vortices condense in the confined phase of the SU(3) vacuum.

To study the vacuum structure of the lattice gauge theories we introduced [1,2] a gauge invariant effective action for the external static background field $\vec{A}^{\rm ext}(\vec{x})$

$$\Gamma[\vec{A}^{\rm ext}] = -\frac{1}{L_4} \ln \left\{ \frac{\mathcal{Z}[\vec{A}^{\rm ext}]}{\mathcal{Z}[0]} \right\} \quad , \quad \mathcal{Z}[U^{\rm ext}_\mu] = \int_{U_k(x)|_{x_4=0}=U^{\rm ext}_k(x)} \mathcal{D}U \; e^{-S_W} \; . \tag{1}$$

$\mathcal{Z}[U^{\rm ext}_\mu]$ is the lattice Schrödinger functional (invariant, by definition, for lattice gauge transformations of the external links), $U^{\rm ext}_k(x)$ is the lattice version of the external continuum gauge field $\vec{A}^{\rm ext}(x) = \vec{A}^{\rm ext}_a(x)\lambda_a/2$, and S_W is the standard Wilson action. $\mathcal{Z}[0]$ is the lattice Schrödinger functional with $\vec{A}^{\rm ext} = 0$ ($U^{\rm ext}_\mu = \mathbf{1}$).

At finite temperature we introduced the thermal partition function in presence of a given static background field:

$$\mathcal{Z}_T\left[\vec{A}^{\rm ext}\right] = \int_{U_k(\beta_T,\vec{x})=U_k(0,\vec{x})=U^{\rm ext}_k(\vec{x})} \mathcal{D}U \, e^{-S_W} \; , \; \beta_t = L_4 = \frac{1}{aT} \, . \tag{2}$$

If we send the physical temperature to zero the thermal functional Eq. (2) reduces to the zero-temperature Schrödinger functional given in Eq. (1).

We would like to detect vortex condensation. To this purpose we use our lattice effective action to define a disorder parameter [3,4] with non vanishing vacuum expectation value in the confined phase:

$$\mu = e^{-F_{\rm vort}/T_{\rm phys}} = \frac{\mathcal{Z}_T[\rm vort]}{\mathcal{Z}_T[0]} \, , \tag{3}$$

where $\mathcal{Z}_T[\rm vort]$ and $\mathcal{Z}_T[0]$ are, respectively, the thermal partition function with a vortex background field and without the background field, $F_{\rm vort}$ is the free energy to create a vortex.

In SU(3) gauge theory we may consider two independent kinds of Abelian vortices T_3 and T_8 (and their linear combinations) associated respectively to the generators λ_3 and λ_8 (for details see [5]).

In case of center vortices the thermal partition function $\mathcal{Z}_T[\mathcal{P}_{\mu\nu}]$ is defined by multiplying by the center element $\exp(i2\pi/3)$ the set $\mathcal{P}_{\mu\nu}$ of plaquettes [6,7] $\mathcal{P}_{\mu\nu}(x_1,x_2,x_3,x_4)$ with $(\mu,\nu)=(4,2)$, $x_4=x_4^\star$, $x_2=\frac{L_s}{2}$ and $L_s^{\rm min}\leq x_{1,3}\leq L_s^{\rm max}$, with L_s the lattice spatial linear size.

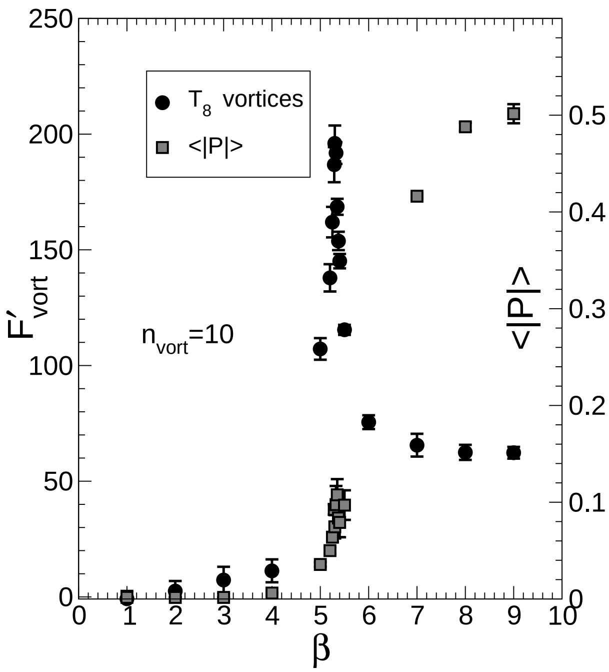

Figure 1. $F'_{\rm vort}$, the β-derivative of the free energy $F_{\rm vort}$ for T_8 Abelian vortices display a peak in correspondence of the rise of the absolute value of the Polyakov loop.

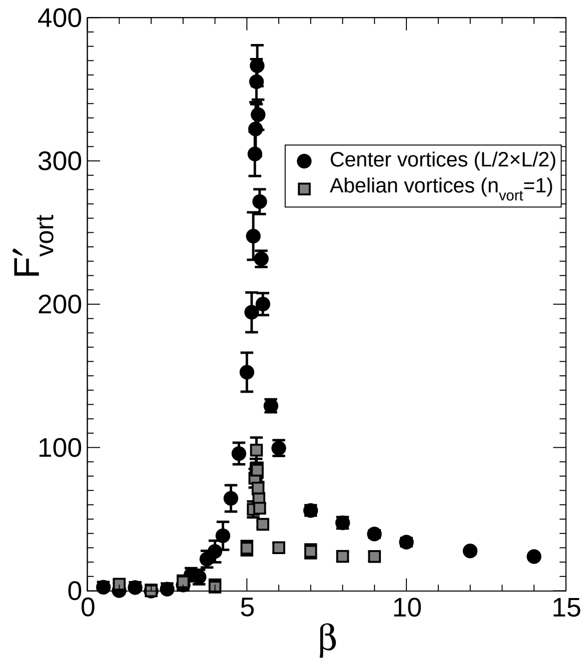

Figure 2. $F'_{\rm vort}$ for center vortices ($L_s^{\rm min}=L/2$) and T_8 Abelian vortices (vortex charge $n_{\rm vort}=1$).

By numerical integration of $F'_{\rm vort}$ we can compute $F_{\rm vort}$ and the disorder parameter μ (see Eq. (3)). Our numerical results suggest that in the confined phase of SU(3) lattice gauge theory $F_{\rm vort}=0$ (in the thermodynamic limit).

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